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The Irreducibility of the Indexical ‘I’

There are various kinds of context-sensitive expressions in a natural language. The most significant ones are the demonstratives like ‘this’, ‘that’ and indexicals like ‘I’, ‘now’, and ‘here’. Although they share the feature of context-sensitivity, I believe there still exist important similarities and differences to be explored. As a preliminary step, in this thesis I will focus on the indexical ‘I’.

H. N. Castañeda is one philosopher who examines the logic of the indexical ‘I’. He does so closely and precisely in an attempt to make it as clear and as distinct as possible. One important argument that Castañeda presents us with concerning the indexical ‘I’ can be called the ‘irreducible thesis of the indexical ‘I’’ ([IT]). Briefly put, he proposes that it is impossible to reduce ‘I’ to other expressions that do not include it. I will present an argument in defense of [IT].

I will develop this thesis as follows. First, based on Castañeda’s argument, I will confirm briefly that knowledge that contains the indexical ‘I’ cannot be reduced to knowledge \textit{de dicto} (§I). Next, I will introduce J. Hintikka’s epistemic logic, which attempts to reduce knowledge that contains ‘I’ to knowledge \textit{de re} and examine Castañeda’s criticism of Hintikka’s argument (§II). In this section I will complement and modify Castañeda’s argument against Hintikka to demonstrate that knowledge \textit{de re} can be neither a sufficient nor a necessary condition for knowledge which contains ‘I’. In the following section I will explore a peculiar kind of knowledge \textit{de re}, which remains outside of the dispute between Castañeda and Hintikka (§III). I would like to show that it is impossible to reduce the Indexical ‘I’ to this type of knowledge \textit{de re}.

Finally, based on the foregoing argument regarding [IT], I will conclude that the indexical ‘I’ has no referent; the indexical ‘I’ cannot be a referring expression\footnote{It is G. E. M. Anscombe who advocates this thesis most clearly. ‘“I” is neither a name nor another kind of expression whose logical role is to make a reference, at all.’ ([1], p.60). In this thesis I will not examine explicitly the relation between Anscombe’ argument and mine about the indexical ‘I’. But here I want briefly to point out two differences between them. The first lies in the procedure by which the thesis is proved (whether using Castañeda’s analysis or not). The second consists in the relation between the thesis and the Cartesian cogito. Anscombe criticizes this but I attempt to defend it, partially. I will indicate this in the last section.}. In my view, this paradoxical conclusion provides a basis for our common linguistic usage of the indexical ‘I’. The indexical ‘I’ can come into existence only when it accompanies this paradoxical
result, and without necessitating this conclusion the indexical ‘I’ can never arise.

§I Castañedas’ Irreducibility Thesis

Castañeda does not examine directly the indexical ‘I’ but, instead, investigates the third-person statement, which attributes self-knowledge to someone.

(1) The Editor of Soul knows that he (himself) is a millionaire.

He introduces a notation ‘he*’, which means the third-person pronoun ‘he himself,’ which attributes self-knowledge to the editor of Soul in (1). ‘He*’ is characterized by following features. (i) ‘He*’ does not represent an indexical reference made by the speaker of the statement. For example, if both (1) and

(2) Anthony knows that I am a millionaire.

are uttered by Privatus, then the indexical reference of ‘he*’ in (1) is not made by Privatus, whereas the indexical ‘I’ in (2), which refers to Privatus himself, is made by himself. (Of course, it is Privatus who uses the token of ‘he* = he himself’ in (1).) (ii) ‘He*’ appears in oratio obliqua. (iii) ‘He*’ has an antecedent, ‘the editor of Soul’, to which it refers. (iv) The antecedent is placed outside of the oratio obliqua. (v) ‘He*’ is used to attribute to ‘the editor of Soul’ a so-called an implicit indexical reference. If the editor were to assert what he knows, he would say that

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2 Castañeda argues for the irreducibility thesis most vigorously in [4]. But my summarization here mainly depends on [7] pp.439-444, where it is more concisely described. In this thesis, following his notation, I will use single quotes,‘’, around a sentence to indicate the name of the sentence, and double quotes,«», around it to indicate the name of a statement formulated with the sentence. I will employ a numeral within parentheses to refer either to the sentence that follows the numeral on its first appearance or to the statement formulated with that sentence, depending on the context.

3 The following argument is applicable to such propositional attitudes as ‘believe’ ‘guess’ ‘say’ ‘argue’ etc., in general, with some modifications. But in this thesis I will focus solely on the verb ‘to know’ since the argument between Castañeda and Hintikka that will be examined in §II is carried out based on examples of ‘knowing’.

4 I think that the following remark by Wittgenstein is true of Castañeda’s invention of this notation to analyze the logic of the indexical ‘I’. „Ich will sagen: Wenn man eine nicht übersehbare Beweisfigur durch Veränderung der Notation übersehbar macht, dann schaft man erst einen Beweis, wo früher keiner war.“ ([17], Teil III, §2) That is to say, it seems that Castañeda has accomplished a proof only in the sense that he invents a notation with which to analyze the logic of the indexical ‘I’, without using the indexical ‘I’ directly. In this thesis I want to shed light on the efficacy of this notation.
using the first-person pronoun, where Privatus used ‘he*’, in uttering (1).

‘He*’ is clearly distinguished from most indexicals by the features ( i ) and (v)\(^5\). Castañeda calls ‘he*’ quasi-indicator\(^6\) and set out an important thesis.

\[\text{[IT.0](Irreducibility Thesis of ‘he*’)}\]
Quasi-indicator ‘he*’ is unanalyzable by proper names, definite descriptions, demonstratives, indexicals, and quantifiers, and is irreducible to the analysis which is composed of them.

I will briefly confirm the grounds for [IT.0], regarding definite descriptions, proper names, and demonstratives (indexicals) in turn. First, (1) neither entails nor is entailed by

(4) The editor of *Soul* knows that the editor of *Soul* is a millionaire.

which results from substituting the definite description ‘the editor of *Soul*’ for ‘he* (he himself)’ in (1). Let me assume that the editor of *Soul* does not yet know that he* has been appointed to the editorship in a closed meeting of the journal’s editorial board. He has read in the newspaper that the editor is bequeathed 2 million dollars and he thinks that he* is still poor. Under this supposition (4) is true but (1) is false. Moreover, in the same situation the editor might believe he* has 2 million dollars in his bank account while also believing that penniless Dr. Jones is still the editor. Then (1) is true but (4) is false. The same type of argument can be applied to any other definite description that refers to the person who is the editor of *Soul*.

Next let me substitute the editor’s proper name, ‘Gaskon’, for ‘he*’. We will assume there exists one and only one individual who has the proper name ‘Gaskon’ within this (hypothetical) world.

(5) Gaskon knows that he* is a millionaire.
(6) Gaskon knows that Gaskon is a millionaire.

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\(^5\) For example ‘he’, which occurs in knowledge *de re* in §II, shares the features ( ii )( iii ) and ( iv ).

\(^6\) Generally Castañeda uses the word ‘indicator’ to signify both ‘demonstratives’ and ‘indexicals’ (cf., [5]).

\(^7\) I omit the attempt to analyze ‘he*’ in terms of the quantifier referred to in [IT.0], because it is not very important. Castañeda briefly examines it as a candidate (C) in [4], pp.132-134. I will examine the possibility of analysis in terms of the quantifier involved in knowledge *de re* in detail in the next section.
In this case we can suppose that Gaskon has forgotten he* is Gaskon, due to amnesia. If Gaskon knows that he* owns enormous property, then under this assumption (5) is true but (6) is false. Under the same assumption, if he realizes that the person whose proper name is ‘Gaskon’ has acquired great wealth but he believes that he* is poor as ever, then (6) is true but (5) is false.

Let us then replace ‘he*’ in (5) with ‘this (this man)’.

(7) Gaskon knows that this (this man) is a millionaire.

We have to be attentive to the fact that it is not Gaskon, the subject of (7), but the speaker of (7), for example Privatus, who makes a reference by using ‘this (this man)’. It is therefore possible that Gaskon does not know that the person to whom the utterer of (7) refers is identical with he*. Then, it is possible that (5) is true though (7) is false, and vice versa. The same pattern of argument is applicable to the other demonstratives and indexicals that can be distinguished from quasi-indicators.

I can formulate the foregoing argument about ‘he*’ in the following way.

[IT.1] ( i ) There is no individual constant ‘a’ such that «The editor of Soul knows that he* is a millionaire» entails its corresponding statement «The editor of Soul knows that a is a millionaire» and ( ii ) the former is entailed by the latter.

§II Defense of Irreducibility Thesis – Criticism of J. Hintikka’s Theory

2-1. From knowledge de dicto to knowledge de re

Even if [IT.1] is accepted, there seems to be room for criticism of the irreducibility thesis from a different point of view. It is the view that regards the object of propositional attitudes including ‘know’ not as de dicto but as de re. For example, (6) can be interpreted in at least two ways. The first interpretation of (6) is that Gaskon knows what the sentence ‘Gaskon is a millionaire’ means. To put it in Frege’s term, Gaskon knows the Gedanke of the sentence and its being true. The second is that Gaskon knows that ‘~ is a millionaire’ is predicates of the person Gaskon, no matter by what individual constants he is referred. In order to distinguish

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8 Castañeda calls ‘individual constants’ the definite descriptions, proper names, demonstratives and indexicals that are clearly distinguished from quasi-indicators. In this thesis I will use the term ‘individual constants’ in that sense.
this from the former I will express the latter interpretation as follows\(^9\).

(8) Gaskon knows of Gaskon that he is a millionaire.

Knowledge \textit{de re} is characterized by the remarkable feature that the description through which \textit{de re} (Gaskon) is referred is indeterminate. Assuming that the referent of the proper name ‘Gaskon’ is identical with that of the definite description ‘the editor of \textit{Soul}’, the knowledge \textit{de dicto} that is attributed to Gaskon in (6) is different from that in

(9) Gaskon knows that the editor of \textit{Soul} is a millionaire.

On the other hand, the knowledge \textit{de re}, which is attributed to Gaskon in (8), is the same as that in

(10) Gaskon knows of the editor of \textit{Soul} that he is a millionaire.

This is because they are indistinctive in that the description by which Gaskon refers to Gaskon (= the editor of \textit{Soul}) is indeterminate.

Within the framework of knowledge \textit{de dicto}, we cannot deny the irreducibility of ‘he\(^*\)’. Because it is necessarily possible to assume that Gaskon might not know he\(^*\) is \textit{a}, it is impossible to substitute ‘he\(^*\)’ for ‘\textit{a}’ in knowledge \textit{de dicto}. However, by introducing the concept of knowledge \textit{de re}, we can avoid this difficulty. In the framework of knowledge \textit{de re} we cannot identify the individual constant ‘\textit{a}’ whose substitutability for ‘he\(^*\)’ we would like to explore. In any case, it at least seems evident that (5) entails (8), while (5) entails neither (6) nor (9) and is entailed by neither (6) nor (9). Therefore, I will examine the reducibility of knowledge about ‘he\(^*\)’ to knowledge \textit{de re}.

2-2. Hintikka’s theory of analyzability\(^{10}\)

Hintikka attempts to reduce ‘he\(^*\)’ to knowledge \textit{de re} in his formal system of epistemic logic. He sets down a first-order functional calculus with individual constants and introduces an operator \langle Ka \rangle. It is to be read as ‘\textit{a} knows’. \langle Ka(p) \rangle is to be read as ‘\textit{a} knows p’ where \langle p \rangle

\(^{9}\) For this type of formulation of knowledge \textit{de re}, refer to [16], pp.9-12, for example. The candidate (F), which Castañeda explores in [4], pp.135-137, corresponds to the reducibility of ‘he\(^*\)’ in terms of knowledge \textit{de re}, although he does not use such an expression.

\(^{10}\) The argument in this section is mainly based on the analysis and dispute in Castañeda [3],[6],[7]and Hintikka [10], [11]. The term is based on Castañeda’s revision of Hintikka’s notation to fit it to their argument.
expresses a sentence with no free variables. Hintikka’s analyzability thesis can be formulated by

$$\text{[AT]} \text{«a knows that he}_{\ast} \text{ at ...» is fully analyzable as «( \exists x) (x = a \& Ka (\ldots x\ldots))»}^{11}.$$ 

He proposes two possible interpretations of the existentially quantified statement in [AT]. Given that \(f(x)\) represents a function with free variable ‘\(x\)’, which ranges over persons as an object domain, the two interpretations are as follows.

(A) \(\exists x \text{Ka } f(x)\) is to be read as «a knows who is \(f\)».
(B) \(\exists x \text{Ka } f(x)\) is to be read as «There is a person known to \(a\) whom \(a\) knows to be \(f\)».

2-3. Castañeda’s criticism (1)

Castañeda attempts to criticize these two interpretations of [AT]. However, in my view, it becomes easier to comprehend the essence of Castañeda’s criticism of [AT] when it is reformulated in a more definite way.

$$\text{[AT’]} \text{«a knows that he}_{\ast} \text{ at ...» if and only if «( \exists x) (x = a \& Ka (\ldots x\ldots))»}.$$ 

Each of Castañeda’s criticisms of [AT] targets one of the conditionals in [AT’]. Initially, I will examine his criticism of

$$\text{[AT’1]} \text{«( \exists x) (x = a \& Ka (\ldots x\ldots))» → «a knows that he}_{\ast} \text{ at ...»}$$

In order to criticize [AT’1], Castañeda introduces a person called ‘Quintus’\(^{12}\). Quintus is brought unconscious to a military tent and suffers from amnesia upon regaining consciousness. Several months after this incident, he becomes a hero in battle. Then, he completely forgets the time during which he has lived as a soldier. Quintus investigates all articles about the war hero to discover he, not he* but the hero, is the only person ever to be wounded 100 times in battle. Being fascinated by this achievement Quintus writes the most authoritative biography of the war hero (guided by a mysterious drive, which is unknown,

\(^{11}\) ‘He}_{\ast} \text{ at ...» in [AT] expresses ‘he*’ of degree1, which has an expression ‘a’ as its direct antecedent. Castañeda introduces the concept of the ‘degree of ‘he*’ to make explicit which subject is the antecedent of ‘he*’ when more than two propositional attitudes ‘to know that’ occur in a sentence. However, since all of the examples that appear in this thesis belong to the class of degree1, neglecting the number won’t affect the substance of the argument.

\(^{12}\) Cf., [7], p.446.
even to himself).

The example can be expressed in terms of the formula [AT’1] in the following way.

\[(11) (\exists x) (x = \text{hero} \& K_{\text{hero}}(x = \text{hero})) \rightarrow \text{The war hero knows that he* is the war hero}^{13}.\]

Based on the interpretation (A), (11) is to be read as

There is a person who is identical with the war hero and whom the war hero knows to be the war hero. \(\rightarrow\) The war hero knows that he* is the war hero.

However, Castañeda says about Quintus that, ‘For most normal situations, regardless of shifts in the criteria for identifying a person, Quintus knows who the war hero was much better than most people, even though Quintus does not know that he* is the war hero.’ \(^{14}\) In short, Castañeda regards the case of Quintus as an evident counter example to (11).

In response, Hintikka raises the objection that the criteria of knowledge (‘know’) that Castañeda assumes is much too weak, and that if it is strengthened he can defend [AT’1]. ‘Castañeda is presupposing a [weak] standard of knowing who \(h\) is, on which one can know who [individual constant] \(h\) is without being able to locate him in the actual world.’ \(^{15}\) ‘The implication [from the antecedent to the consequent in [AT’1] ] would hold if \(h\)’s knowing who \(h\) is gave him the ability to say of an actual flesh-and-blood individual that he is \(h\).’ \(^{16}\) ([…]added, quoter). Judging from these descriptions, the ‘strong’ criterion of knowing who \(h\) is seems to be ‘knowledge which enables one to locate \(h\) as an actual flesh-and-blood individual in the actual world.’

Of course, Castañeda replies that Quintus obviously satisfies the ‘strong’ criterion that Hintikka demands because ‘Quintus by his careful study of the actual war hero’s actual exploits has acquired the ability to say of an actual flesh-and-blood individual that he (that individual, not he himself [he*]) is …the war hero.’ \(^{17}\)

2-4. Reinforcement of Castañeda’s criticism (1)

The previous section exhausts almost all disagreements between Castañeda and Hintikka

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13 An identity statement whose one term is the epistemic subject (hero) appears in the scope of K in the antecedent of (11). This makes the content of recognition in the antecedent much richer than in the original formula [AT’1]. Therefore, (11) means a more favorable condition for those who will defend [AT’1] and a less favorable condition for those who will criticize [AT’1]. Since here I intend to criticize [AT’1], this alteration won’t undermine the validity of the argument that follows.
14 [7], p.446.
15 [11], p.53.
17 [7], p.454.
regarding [AT’1]. While I think that Castañeda’s criticism is valid, the criterion of ‘strong’ knowledge demanded by Hintikka is too ambiguous for Castañeda’s criticism, based on it, to be conclusive. For, if the criterion is strengthened by adding some condition, it will still allow Hintikka to argue against Castañeda that he does not fulfill it. In order to rule out such counterarguments I will introduce the idea of the logically ‘strongest’ criterion of knowledge within the constraints imposed on the argument.

Let me assume that Quintus knows of Quintus (= the war hero who is an actual flesh-and-blood individual) and all the correct facts that Quintus has known and all correct events that have occurred to him up to the present. Quintus knows of him that he was brought unconscious to a military tent and suffered from amnesia on regaining consciousness, and that he then became a hero in battle and completely forgot his time in the army, before investigating all articles about the war hero to discover that the hero was the only person ever to be wounded 100 times in battle. Even under the supposition of (empirically impossible) omniscience about Quintus it is logically possible that Quintus does not know he* is Quintus = the war hero. Quintus comprehends all information about Quintus accurately by reading a daily newspaper, “Quintus Times”. Its articles are based on images documenting all of Quintus’s behavior, taken by mobile cameras tracking Quintus. Almost all of Quintus’s beliefs about who he* is and has been, what he* has done, and how he* has lead his life by that time are false. Of course, since he is self-conscious Quintus knows what he* is doing when he* does it. It seems that Quintus ought to notice that he* cannot be any other person but Quintus, based on the perfect coincidence between the knowledge of Quintus that he obtains by reading the “Quintus Times” every day and that of him*. However, for some reason Quintus is peculiarly composed, such that he is able to comprehend correctly what he* is doing in the instant, but his memory of it is transmuted into a false one after a little time has passed. He nevertheless retains all correct information about Quintus that he acquires from the “Quintus Times”. Under this assumption, although Quintus is given the ability to locate an actual flesh-and-blood individual Quintus in the actual world accurately, through accumulating the maximum amount of information about Quintus, he can never come to know that he* is Quintus.

Such an imagined situation could not be realized empirically, but at least the supposition is logically consistent. From this I can conclude that even if the antecedent of (11) ([AT’1]), with a criterion of knowledge that is as strong as is logically possible, is satisfied, the consequent does not necessarily follow.

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18 It can be said that this is one variation of Lewis’s supposition of two omniscient Gods that lack knowledge distinguishing which one is he himself (cf., [14], pp.520-521).
2-5. Castañeda’s criticism (2)
Now I will examine Castañeda’s criticism of the other conditional

\[\text{[AT’2]} \, \langle a \text{ knows that } he^* \text{ is } \ldots \rangle \rightarrow \langle (\exists x) \,(x = a \& Ka (\ldots x\ldots)) \rangle\]

Castañeda plainly admits that if an existential exemplification \(h = a \& Ka (\ldots h\ldots)\) is admitted for an individual constant \(h\) whose referent is the same person that \(a\) refers to, it will contradict (the general formula of) [IT.1]. Assume ‘a’ is ‘the editor of Soul’ and that the individual constant ‘h’, which refers to the same person that ‘a’ refers to, is ‘Gaskon’. If this existential exemplification is permitted it will follow that

\[\langle \text{The editor of Soul knows that } he^* \text{ is a millionaire} \rangle \text{ entails } \langle \text{The editor of Soul knows that Gaskon is a millionaire} \rangle.\]

This statement contradicts [IT.1]. Therefore, [AT’2] can only be criticized if the calculus of quantification in an existential logic is permitted within Hintikka’s epistemic logic. However, Castañeda concedes to Hintikka that there is a proper reason why the existential exemplification is restricted within the system of epistemic logic. ‘[I]n pure quantification we must have the analogue of … “(∃x)(x is f)” implies, for some constant ‘h’, “h is f”. But in the case of epistemic logic, we cannot invent names or descriptions to instantiate ‘(∃x)’. For instance, we cannot invent a name for the object that satisfies “(∃x)(Ka(x is f))” and put “Ka(h is f)”; for then we would be claiming that a knows that object under the name ‘h’, which would be outrageous. In short, existential instantiation needs a special treatment in epistemic logic.’

Castañeda concedes to Hintikka this restriction on existential exemplification in his epistemic logic. Presupposing this concession, Castañeda attempts to criticize [AT’2] by deriving a contradiction from [AT’2] within Hintikka’s system. His argument can be summarized as follows.

1. [AT’2] is valid in a sense.

For example,

(12) The war hero knows (at time t) that he* was wounded100 times.

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\(^{19}\)[7], p.452.
entails

\[(13) \ (\exists x)(x = \text{hero} \ & \ K_{\text{hero}}(x \text{ was wounded 100 times}))\]

In interpretation (B), (13) is to be read as, ‘There is a person who is identical to the hero and is known to the hero such that the hero knows that at time t that person was wounded 100 times.’ Indeed, this entailment seems to be intuitively true.

2 In epistemic logic an existentially quantified statement is not permitted to be exemplified in terms of an individual constant.

Consequently [AT’2] cannot be criticized on the basis that an exemplification of (13) in terms of an individual constant brings in a contradiction with [IT.1]. It contradicts [IT.1] that (12) entails an exemplification of (13), for example «Quintus = hero \ & \ K_{\text{hero}} (\text{Quintus was wounded 100 times})».

3 The rule (C. ind=) is prescribed in Hintikka’s system.

For example, assuming ‘b’, ‘c’ as individual constants, (C. ind=) prescribes that when a model set \( \mu \) includes

\[(14) \ (\exists x)(b = x \ & \ K_b(b = x))\]
\[(15) \ (\exists x)(c = x \ & \ K_c(c = x))\]
\[(16) b = c\]

then

\[(17) K_b(b = c)\]

is also included in \( \mu \). This rule (C. ind=) can be put as ‘the premises (14), (15), (16) entail (17)’20. Castañeda contends that [AT’2] and (C. ind=) therefore entail a contradiction. Since he does not show the procedure explicitly, let me demonstrate it. First, he points out that (12) (the antecedent of [AT’2]) and

\[(18) \ (\exists x)(x = \text{hero} \ & \ \neg K_{\text{hero}}(x \text{ was wounded 100 times}))\]

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20 Castañeda reads the other rule in Hintikka’s system in the same way as this (cf., [7], p.450-451).
are compatible. Under interpretation (B), (18) is to be read as, ‘There is a person who is identical to the hero and is known to the hero, such that the hero fails to know at time t that this person was wounded 100 times.’ This person might, for example, be the biographer of the hero. The biographer happens to be identical to the hero and is an actual person known to the hero, but the hero does not know that he (the biographer) was wounded 100 times.

Let us assume that (12) is compatible with (18), following Castañeda’s suggestion. Then (since ‘the war hero’ is synonymous with ‘the person who was wounded 100 times’ here) (12) implies

\[(14)' (\exists x)(\text{hero} = x \& K_{\text{hero}}(\text{hero} = x))\]

By (18), there is a person who is identical to the biographer and is known to the hero such that the hero knows this person is the biographer:

\[(15)' (\exists x)(\text{biographer} = x \& K_{\text{hero}}(\text{biographer} = x))\]

And as a matter of fact,

\[(16)' \text{hero} = \text{biographer}\]

From premises (14)', (15) ', (16) ', and (C. ind=), it follows that

\[(17)' K_{\text{hero}}(\text{hero} = \text{biographer})\]

However (17)' contradicts (18), which is described above. Indeed, according to (18) the hero does not know the biographer was wounded 100 times (as the war hero). Thus, even if he concedes to Hintikka the existential exemplification within the consequent in [AT'2], Castañeda can draw a contradiction from [AT'2] within Hintikka’s epistemic logic. This is the basis upon which Castañeda criticizes [AT'2].

2-6. Examination on Castañeda’s criticism (2)

Is Castañeda’s criticism of [AT’2] appropriate? In my view, it is not an appropriate criticism of [AT’2] although it might be one of Hintikka’s system or, more strictly, of the rule (C. ind=). The same type of inference as was described above, which leads to a contradiction, can be applied just as well to the ordinary statements that have nothing to do with self-consciousness. Given that ‘a’ = ‘hero’, ‘b’ = ‘Hesperus’, ‘c’ = ‘Phosphorus’, from following
premises

(14)" (∃x)( Hesperus = x & K_{hero}(Hesperus = x))
(15)" (∃x)( Phosphorus = x & K_{hero}(Phosphorus = x))
(16)" Hesperus = Phosphorus

and from (C. ind=) it follows that

(17)" K_{hero}(Hesperus = Phosphorus)

This inference contradicts our intuition. It is common for (17)"not to hold while the premises are satisfied. In short, Castañeda’s criticism of [AT’2] is directed at the fact that Hintikka’s system encodes very unnatural concepts of knowledge and belief 21, as is exemplified by the rule (C. ind=). What is more, only if the epistemic ability to satisfy (C. ind=) is presupposed will we avoid a contradiction in the case of the war hero, not to mention that of Hesperus.

In order to satisfy (C. ind=) the epistemic ability is required to know all individual constants (proper names, definite descriptions) that refer to the identical object as belonging to the same class of individual constants that have the same referent. Given this epistemic ability, those who know the object (= Quintus) that is referred to by an individual constant ‘the war hero’ will also know the other individual constant ‘the biographer of the war hero’, which refers to the same object, and vice versa. Presupposing this ability, (17)` will hold without any difficulty. Note that the assumption of the epistemic ability to satisfy (C. ind=) is entirely self-consistent, even if the ability cannot be realized empirically. Consequently I cannot help but conclude that Castañeda’s criticism of [AT’2] is not valid.

2-7. Modification of Castañeda’s criticism (2)

Is it impossible, then, to criticize [AT’2]? If so, there is room for the reducibility of [he*] in a sense of [AT’2]. However, in my view, [AT’2] is not beyond criticism. I think that Castañeda concedes too much to Hintikka in his criticism of [AT’2]. I find it problematic that he admits the validity of the entailment in [AT’2], in a certain sense (①). I will contend that the entailment in [AT’2] is not valid in any sense. Castañeda considers the following instantiation of [AT’2] to be valid:

The war hero knows (at time t) that he* was wounded100 times.
→ (∃x)(x = hero & K_{hero}( x was wounded 100 times))

21 ‘Thus, Hintikka has formalized very strong senses of “know” and “believe,” for which probably there are no human instances.’ ([3], p.133).
For an exact understanding of this conditional it is necessary to define strictly the free variable \( x \) and its object domain. Castañeda defines the free variable \( x \) as ‘ranging over persons’. By defining the object domain of \( x \) as comprising expressions that uniquely refer to a person, we can develop the argument more precisely. Under the definition, for example, the object domain includes Quintus, the war hero, the person who was wounded 100 times, the biographer of the hero, etc., as these are expressions that refer uniquely to the person Quintus. Since they share the same referent, the equation Quintus = the war hero = the wounded 100 times = the biographer of the hero should hold. Let us assume this definition of the object domain. First, according to [IT.1], for any individual constant \( a \) (Quintus, the war hero, the person who was wounded 100 times, the biographer of the hero, …)

\[
\text{The war hero knows (at time } t \text{) that he* was wounded100 times.} \\
\rightarrow a = \text{hero } \& K_{\text{hero}}(a \text{ was wounded 100 times})
\]

does not hold. However, when one accepts [AT’2], the following implication of the disjunctions of exemplifications by individual constants holds.

\[
\text{The war hero knows (at time } t \text{) that he* was wounded100 times (= } w). \\
\rightarrow (a = \text{hero } \& K_{\text{hero}}(w = a)) \lor (b = \text{hero } \& K_{\text{hero}}(w = b)) \lor (c = \text{hero } \& K_{\text{hero}}(w = c)) \\
\lor \ldots
\]

But even when the war hero knows that he* was wounded100 times, it is possible to suppose that the war hero does not know of any expression that uniquely refers to the war hero. He might, for example, have forgotten all expressions that uniquely refer to him (not him* but the hero), because of temporary memory problems, or he might believe erroneously that other proper names or definite descriptions refer to the hero (but not to him*)\(^2\). If we accept that our epistemic ability is finite, it is not problematic to reduce existential quantified statements to disjunctions of finite numbers of propositions. Even if the object domain includes infinitely many members, the criticism described above is still valid\(^3\).

\(^2\) Cf., [7], p.449.

\(^3\) Referring to Castañeda, Perry makes basically the same contention as this one under the phrases ‘the non-necessity of belief’ and ‘the irrelevancy of belief’ (cf., [15], pp.486-487). The case of disordered memory corresponds to the former and that of false belief corresponds to the latter.

\(^4\) In this example, since the content of knowledge is an identity statement, the one term ‘the person who was wounded 100 times = w’ is the expression that designates the hero uniquely. Consequently the antecedent can at least be said to imply the exemplification for \( w \), \( w = \text{hero } \& K_{\text{hero}}(w = w) \). Even though it is accepted, the same type of claim cannot hold when the content of knowledge does not include a uniquely referring expression («The war hero knows that he* is a millionaire»). This implication holds true only for a self-identity statement whose one term is an individual constant that the hero knows contingently refers uniquely to (not he* but) the hero. Even though reducibility of ‘he*’ to such knowledge is admitted, it seems not to have any significance.
Consequently [AT’2] is not valid in any sense, despite Castañeda’s concession. In order to maintain the validity of the implication, it seems that there is no other option but to assign to the object domain a set of objects quite distinct from the one defined above.

§III  Peculiar Kind of Knowledge de re

3-1. Knowledge de re about self based on acquaintance

By the preceding argument, I have criticized [AT’], demonstrating the irreducibility of ‘he*’ to knowledge de re. However, there remains a peculiar kind of knowledge de re that requires detailed exploration, separate from the dispute between Castañeda and Hintikka. It can be called ‘knowledge de re based on acquaintance’\(^{25}\). I think that the following argument will make clear the reason Castañeda does not approach it directly.

I will present an example, originally formulated by J. Perry\(^{26}\), to illustrate ‘knowledge de re about self based on acquaintance’ (which is abbreviated as [RKa] hereafter, while ‘knowledge de re about self based on an individual constant’, scrutinized in the previous section, is denoted [RKc]).

Gaskon notices sugar scattered about the floor of a store. In fact, it is spilling out through a tear in the sac of sugar that Gaskon has put on his cart, but he does not realize this. There is a mirror on the opposite end of the aisle. Gaskon sees that the shopper who is reflected in the mirror is making a mess and approaches the mirror. Gaskon says to himself, ‘He is scattering sugar around.’ He has not yet realized that he himself is reflected in the mirror.

Then it can be said «Gaskon knows of (not him* but) Gaskon that he is making a mess in a store». And it can be supposed that Gaskon does not know of any individual constant that uniquely refers to the person making a mess in the store, who is reflected in the mirror. For example, Gaskon might believe improper individual constants such as ‘the shopper who is making a mess in the store’, or ‘the shopper who is approaching the mirror’. Although he does not know any proper individual constants, we can say that Gaskon acquaints himself with the person making a mess in the shop via the reflection of the mirror. Although it is impossible to express it in terms of individual constants, I can express this knowledge based on acquaintance by using a description that includes ‘he*’. For example, I can write ‘the person whom he* sees through the reflection of the mirror’. A paper by D. Lewis makes

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\(^{25}\) Lewis classifies knowledge de re into seven types (although he argues not about knowledge but about belief). According to his view, they share ‘a relation of acquaintance’ (cf., [14], pp. 541-542). But ‘knowledge de re based on acquaintance’ in this thesis has a narrower extension than Lewis’s and seems to be contained within part of the types (type(1), (5)).

\(^{26}\) Cf., [16], p.12. I have changed Perry’s example a bit to apply it to the argument in this thesis.
explicit this feature of [RKa]. ‘[W]atching is a relation of acquaintance. I watch myself in reflecting glass, unaware that I am watching myself. I ascribe to myself, under the description “the one I am watching,” the property of wearing pants that are on fire. I therefore believe de re of the one I am watching – that is, myself – that his pants are on fire.’

Let me formulate the conditions for [RKa] to occur, based on the preceding argument.

[ Conditions of [RKa] ]
① About a subject a «a knows of a that he is…» holds. («Gaskon knows of (not him* but) Gaskon that he is making a mess in a store»)
② Subject a does not know any individual constant that refers uniquely to a.
③ Subject a knows a through the relation of acquaintance. Therefore a can refer to a by the definite description that includes ‘he*’. («Gaskon knows of the person (= Gaskon) whom he* sees through the reflection of the mirror in the shop that he is making a mess»)

This formulation of [RKa] makes evident the reason that Castañeda does not directly deal with [RKa] in his argument regarding the irreducibility of ‘he*’. For, as condition ③ shows explicitly, even if knowledge about ‘he*’ (which is abbreviated as [K-he*] hereafter) is reduced to [RKa], ‘he*’ cannot be analyzed away by the reduction.

As is shown explicitly in the examples of Perry and Lewis, [RKa] cannot be a sufficient condition for [K-he*] to come into existence. Gaskon knows of Gaskon that he is making a mess in the shop through a relation of acquaintance, satisfying the conditions of [RKa] without knowing that he* is making a mess. Alternatively, Gaskon knows of Gaskon that he is going to be burned through a relation of acquaintance, satisfying the conditions of [RKa] without knowing that he* is going to be burned.

Furthermore, I will demonstrate that [RKa] cannot be a necessary condition for [K-he*] to occur. If this claim is valid, I can conclude that [K-he*] cannot be reduced to [RKa] – ‘knowledge de re about self based on acquaintance’ – in any sense. I will prove it by demonstrating that [RKa] can be reduced to [RKc] – ‘knowledge de re about self based on an individual constant’ – while [K-he*] cannot be.

3-2. Reducibility of [RKa]
G. E. M. Anscombe has invented an interesting language-game, which is useful for exploring [RKa]. In the language-game each member of a community is labeled with two names. One name appears on the back and at the top of the chest of an individual; the bearer can see neither. These names are various and differ from person to person, ‘B’, ‘C’, …, ‘Z’. The other name, ‘A’, is stamped on the insides of their wrists and is the same for everyone. In making

27 [14], p.543.
28 Cf., [1], p.49.
reports on the actions of other people, each person uses the name on the other person’s chest or back, if it is visible. Each person has also learned to respond to the utterance of the name on his own chest and back in exactly the way that we tend to respond to utterances of our proper names. Reports on one’s own action are made using the name on the wrist. Such reports are made, however, not only on the basis of observation, but also on that of inference and testimony or other information. For example, B might derive conclusions that are expressed by sentences with ‘A’ as the subject, from other people’s statements, which use ‘B’ as the subject.

This ‘A’ is designed by Anscombe for the purpose of sharing important features with the indexical ‘I’. The common features are ① everyone is allowed to use it, but ② everyone, except for the user himself, is prohibited from using it to designate the user. For example, anyone but B cannot use it (‘A’ or ‘I’) for the purpose of designating B.

I want to revise this game in order to make it more useful in what follows. Anscombe assumes that people in the language-game community lack self-consciousness. In contrast, I will suppose that they are self-conscious beings. In other words, they can use the indexical ‘I’ (or ‘he*’) in the same way as we do, with one important restriction. They are conditioned not to know who he* is – they do not to know whether he* is B, or C,..., or X, though they are able to use the indexical ‘I’. This impediment is caused primarily by their being deprived of a sense of freedom, which usually accompanies physical movements. Specifically, they cannot perceive whose limbs or mouths they move, i.e. whether it is B’s limbs or mouth or C’s or D’s or ..., Z’s that they move. They are deprived of the sense necessary for discriminating it. From the other person’s point of view, for instance, B appears to speak by moving B’s mouth at will and conducts various actions by moving B’s limbs at will. However, it is felt by B that he can move neither B’s mouth nor B’s limbs at will, and that they move independently of B’s will when they move.

Even under such conditions, [RKa] is achievable. For example, G can acquire various kinds of knowledge of the person on whose wrists ‘A’ is inscribed based on acquaintance, without knowing any individual constant that uniquely designates G. G might know that this person is making a mess in a store. Then, it is certain that the three conditions of [RKa] are satisfied.

① About a subject G «G knows of G that he is making a mess in a store» holds.
② Subject G does not know any individual constant that refers uniquely to G.

29 ‘[I]nstead of these rather inhuman people, we suppose machines that are equipped with scanning devices, are marked with signs in the same way as the people in my story were marked with their names, and are programmed to translate what appears on the screen of their scanners into reports’ ([1], p.49-50).
Subject G knows G through a relation of acquaintance. Therefore G can refer to G by the definite description, which includes ‘he*’. («G knows of the person (= G), on whose wrists he* recognizes the sign A is stamped, who is making a mess in a store»)

Within this language-game, we assume that G does not know that he* is G. From G’s perspective, the body of the person on whose wrist G can perceive the sign A is one that G cannot move at will, just as he cannot move B’s body, or C’s, or D’s. G can observe G’s body only as being in the same alignment as this other person’s (distinguished only by the fact that, for some reason or another, he cannot observe the chest or back of this person).

Using this example, I will demonstrate that [RKa] can be reduced to [RKc] (‘knowledge de re about self based on an individual constant’). When G exchanges information regarding the person he* refers to by using the sign A (=G), he will come to know that ‘G’ is inscribed on the person’s back and chest, which he* cannot see. Then, he will also discover that the description, ‘the person on whose wrist he* recognizes sign A is stamped’, is interchangeable with the description g, ‘the person on whose back and chest sign G is stamped’. This recognition transforms the content of G’s knowledge from [RKa] into [RKc]. Because G comes to know a proper individual constant that designates G (=g), he can no longer satisfy condition ② of [RKa]. In Hintikka’s notation, the [RKc] which G obtains can be formulated in the following way.

\[ g = G \& K_0 (g = G) \rightarrow (\exists x) (x = G \& K_0 (x = G)) \]

Since the description g is an expression that uniquely designates an individual, it belongs to the object domain of the free variable x. (On the other hand, since the description ‘the person on whose wrist he* recognizes sign A is stamped’ includes ‘he*’ and therefore it alone cannot uniquely designate an individual; it cannot belong to the object domain). The following knowledge has enabled [RKa] to be converted into [RKc].

(19) G knows that the person to whom he* refers by use of the sign A = the person on whose back and chest the sign G is inscribed.

However I cannot claim the reducibility of [RKa] to [RKc] only on the basis of the argument so far. For I have not yet made it clear in what sense [RKa] can be reduced to [RKc]. In order to answer the question let me assume the situation in which G loses the knowledge of (19). Assume that the sign on G’s wrist has worn off and transformed gradually. By the time t₁ it has changed into a form that cannot be identified as ‘A’ whereas the sign G on the back and chest has kept its original form. Then, since G cannot designate anyone by the description ‘the person on whose wrist he* recognizes sign A is stamped’, the equation in (19) no longer
holds. Even in that situation, G can acquire information about G and locate an actual flesh-and-blood individual G accurately in the actual world by using the individual constant \( g \). Even if he has lost the description necessary for satisfying condition ③ of [RKa], G has not lost the means necessary for locating G accurately in the actual world. This means that there is no substantial difference between the situation in which only [RKa] is available to G before the knowledge of (19) comes into existence and the one in which only [RKc] is available to G, after the knowledge of (19) gets lost. This fact captures the reducibility of [RKa] to [RKc].

3-3. Irreducibility of [K-he*] to [RKc]
By constrastong the case of [RKa] with the one of [K-he*], I can elucidate the reducibility of [RKa] to [RKc]. Let us assume that G knows that he* sees someone making a mess in a shop without knowing any individual constant that uniquely refers to G. Then, all at once G has come to know who he* is, that is to say, to know that he* is G. This is because he has suddenly realized whose body he* can move at will. Under the situation

\[
(20) \text{G knows that he*} = \text{the person on whose back and chest sign G is inscribed.}
\]

So long as (20) holds, the information that G knows of G is equivalent to that which G knows of him*. Therefore

\[
(21) \text{G knows that } \varphi(\text{he*}). \iff \text{G knows that } \varphi(\text{G}).
\]

For example,

\[
(22) \text{G knows he* sees someone making a mess in a shop.} \iff \text{G knows G sees someone making a mess in a shop.}
\]

Here the recognition of identity in (20) enables [K-he*] to become [RKc] and vice versa. Just as in the case of [RKa], let me assume that G is deprived of the recognition of (20). Then again he has fallen into a state in which he does not know who he* is. This is true, even under the supposition that an individual constant \( g \) (‘the person on whose back and chest the sign G is inscribed’) refers correctly and uniquely to G, and that by its use G can locate G accurately in the actual world («G knows that G sees someone making a mess in a shop»). G has lost knowledge necessary for locating him* in the actual world.

There is a substantial and essential difference between the case in which the recognition of (20) holds and the one where it does not. By losing the knowledge of (20), G is deprived of the significant relationship between G’s locating he* in the actual world and G’s locating a
flesh-and-blood individual G in the actual world. ‘He*’ ([K-he*]) is not interchangeable with the description $g$ ([RKc]) in the same sense that the description ‘the person on whose wrist he* recognizes the sign A is stamped’ ([RKa]) is. Consequently [K-he*] cannot be reduced to [RKc] in this sense, as quite distinct from [RKa].

3-4. Irreducibility of [K-he*] to [RKa]

Based on the preceding argument, I will confirm that [RKa] cannot even be a necessary condition for [K-he*]. Assume that [K-he*] comes into existence, for example, if G knows that he* is the editor of *Soul*. Even under the condition, ③ of [RKa]

③ Subject $a$ knows $a$ through a relation of acquaintance. Therefore $a$ can refer to $a$ by the definite description, which includes ‘he*’.

is not satisfied. This is because the subject $a$ cannot designate $a$ by the use of ‘he*’. For if it were possible, ‘he*’ would be interchangeable with an individual constant that refers uniquely to $a$, for instance the description $g$. However, I have demonstrated that this cannot occur, in the previous section. (On the other hand, since the description that includes ‘he*’ in [RKa], ‘the person to whom he* refers by the use of sign A’ is interchangeable with an individual constant, it can be regarded as referring to $a$). From the fact that [K-he*] does not satisfy ③ it will follow that [K -he*] does not satisfy the condition ①

① About a subject $a$ «$a$ knows of $a$ that he is...» holds.

This is because the ‘he*’ that occurs in [K-he*] cannot refer to the subject $a$.

Conclusion

In this thesis I have attempted to demonstrate the irreducibility of the indexical ‘I’ (‘he*’) or [K-he*], in three steps. First, [K-he*] is irreducible to knowledge *de dicto*. I confirmed this using Castañeda’s argument (§I ). Second, [K-he*] is irreducible to knowledge *de re* about self based on an (implicit) individual constant ([RKc]) (§II ). I attempted to prove this via a detailed examination of a dispute between Hintikka and Castañeda. More precisely, I demonstrated that [RKc] can be neither a sufficient nor a necessary condition for [K-he*] by complementing and modifying Castañeda’s criticism of Hintikka. And third, [K-he*] is irreducible to knowledge *de re* about self based on acquaintance ([RKa]) (§III ). The fact that the expression of [RKa] must include ‘he*’ by its nature has excluded it from
Castañeda’s argument about the reducibility of [K-he*]. However, I have shown that [RKa] is interchangeable with [RKc] in some remarkable respects. In this sense [RKa] can be said to be reducible to [RKc], while [K-he*] does not have such interchangeability with [RKc]. Therefore, unlike [RKa], [K-he*] is intrinsically distinct from [RKc]. In other words [K-he*] is also irreducible to [RKa].

If the preceding arguments, as summarized above, are valid, I can draw the following thesis from it.

[Thesis I] The indexical ‘I’ (‘he*’) in [K-he*] cannot be a referring expression in any sense.

I think that all claims of reducibility of the indexical ‘I’ (‘he*’), such as those seen in Hintikka’s argument, aim at securing a referent for the indexical ‘I’ (‘he*’) to justify the interpretation of it as a referring expression. Attempts to reduce it to knowledge \textit{de dicto} seek to transform the indexical ‘I’ into referring expressions such as proper names or definite descriptions or demonstratives or indexicals, which are components of \textit{de dicto}. Attempts to reduce it to knowledge \textit{de re}, whether it is [RKa] or [RKc], try to interpret [K-he*] as knowledge \textit{de re}, i.e., of an actual flesh-and-blood individual. These attempts also aim to assimilate the indexical ‘I’ (‘he*’) into a referring expression \textit{de re} (of an actual flesh-and-blood individual), but in a somewhat more roundabout way than in the attempt in terms of knowledge \textit{de dicto}. Since I have criticized all of these attempts, [Thesis I] is supported (in so far as an alternative theory has not been proposed that explains the indexical ‘I’ appropriately as a referring expression).

I will now show additional grounds for [Thesis I], from a substantially different point of view. In our ordinary language-game, one who has been considered as obtaining a mastery of the language is granted a privilege with respect to first-person statements. Let me examine a following example.

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Though there is not enough space in this paper, I will briefly express my thought about Perry’s theory of indexicals on the basis of the preceding argument. On the surface, Perry’s theory agrees with Castañeda’s irreducibility thesis, but as it requires securing a referent for the indexical ‘I’, it must actually contradict the thesis. It seems to me that Perry’s and D. Kaplan’s attempts to decompose the indexical ‘I’ into character and content are simply attempts to reduce ‘I’ to [RKa], and therefore [RKc]. In other words, they are attempts to assimilate ‘I’ into the peculiar kind of proper name ‘A’ that is presented in this thesis, based on Anscombe’s idea. Lewis’s criticism of Perry, which follows, can be interpreted from that point of view. ‘[B]elief \textit{de se} falls under belief \textit{de re}.’ ‘[S]elf-ascription isn’t quite the same thing as ascription, \textit{de re}, to oneself.’ ([14], p.543). According to my view, Lewis’s ‘belief \textit{de se}’ corresponds with [RKc] and [RKa] and his ‘self-ascription’ corresponds with [K-he*]. That is to say, ‘belief \textit{de se}’ ([RKc],[RKa]) is a variant of knowledge \textit{de re}, and is therefore reducible to it, but ‘self-ascription’ ([K-he*]) is irreducible to it.
(23) Gaskon asserts that he* believes that the editor of *Soul* is a millionaire.

In this case there is room for Gaskon to make some mistakes about what he believes, «the editor of *Soul* is a millionaire», whereas he is granted infallibility about his* believing the content itself. However, in any theory that considers ‘he*’ to be a referring expression, there is no room for such infallibility. If ‘he*’ were a referring expression, it could be substituted for another referring expression or its referent. When ‘he*’ is substituted for it, the clause whose subject is ‘he*’ ought to be converted into a complete proposition. For there does not exist any ‘incomplete component’ in the sense of Frege. For example, the result of such a substitution is

(24) Gaskon believes that the editor of *Soul* is a millionaire.

However, obviously (24) cannot be an infallible proposition to anyone, including Gaskon himself. Among others it can be, because there is always room for misidentification of the person who occupies the place of its subject, Gaskon. Even Gaskon might believe it is not he*but some other person who is Gaskon. For exactly the same reason, (23) also cannot be infallible as a whole. Then, how can one express the infallibility peculiar to the first person statement? It is only in this way: «when Gaskon asserts that he* believes that the editor of *Soul* is a millionaire, it is only his*believing that is infallible». It is this form of statement that prevents ‘he*’ from being substituted for any referring expression. [Thesis I], which states that ‘he*’ is not a referring expression, gives grounds for the fact that the clause that Gaskon asserts cannot be a complete proposition by itself, which allows room for infallibility. From this point of view, [Thesis I] and the irreducibility of the indexical ‘I’ that serves as grounds for [Thesis I] do not so much contradict as provide a basis for our linguistic practice of the infallibility of first person statements.

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31 Even in this case, of course, Gaskon is not exempt from an error arising from the meaning of words he uses. Under this reservation the infallibility admitted here can be said to be essentially the same as the absolute certainty of the Cartesian cogito.

32 In[4] Castañeda insists explicitly as a thesis (H*I) that ‘he*’ is ‘essentially incomplete’ in that sense. ‘The pronoun ‘he*’ is strictly a subordinate pronoun: it is by itself an incomplete, or syncategorematic, symbol, and every sentence or clause containing a token of ‘he*’ which is not in oratio oblique, is also an incomplete or syncategorematic sentence or clause.’ ([4], p.151).
References